Frank P. Ramsey: Partial Belief and Subjective Probability

In 'Truth and Probability' (1931a), Ramsey sets out an influential account of the nature, measurement, and norms of partial belief. The essay is a pioneering work on *subjectivist* interpretations of probability (also known as *personalist* interpretations; see the entry on Interpretations of Probability). According to subjectivism, probabilities can be interpreted as numerical representations of an individual's subjective degrees of confidence, such that two individuals could assign different probabilities to the same proposition even given the same evidence. First written in 1926 and still incomplete at the time of its publication in 1931 (a year after Ramsey's death), 'Truth and Probability' was not very widely discussed until after the publication of Savage's *The Foundations of Statistics* (1954), at which time subjectivism came to prominence. Many of the key ideas and arguments original to the essay have reappeared in later foundational works in the subjectivist tradition (e.g., de Finetti 1937, Savage 1954, Jeffrey 1965). This section outlines some of Ramsey's major contributions to this tradition.

Ramsey's central goal in 'Truth and Probability' is to show that the laws of probability provide us with a 'logic of partial belief'—that is, the laws specify general necessary conditions on any consistent set of partial beliefs, in a manner analogous to how the laws of classical logic might be taken to generate necessary conditions on any consistent set of full beliefs. The argument for this claim is grounded in Ramsey's novel approach to probability, namely subjectivism. The essay begins, however, with a discussion of the two major approaches to understanding probabilities at the time in Cambridge: frequentism, and a version of the logical interpretation put forward by the economist John Maynard Keynes in *A Treatise on Probability* (1921).

Regarding frequentism—according to which the probability of an event is the relative frequency with which that type of event occurs, or would occur, over repeated trials—Ramsey adopts a conciliatory tone. For many cases (e.g., coin flips and rolls of a die), frequencies provide a natural interpretation of the probability calculus, but they are insufficiently general for Ramsey's purposes. (We'll return to the role of frequencies in Ramsey's theory below.) On the other hand, Ramsey presents a detailed critique of Keynes' theory. According to Keynes, probabilities are an objective and quantifiable relation between propositions—roughly, the probability of an hypothesis h, given evidence e, is the degree to which h is logically implied by e. Importantly, Keynes' assumed that this relation could be perceived through intuition—to which Ramsey objects:

> [T]here really do not seem to be any such things as the probability relations [Keynes] describes. He supposes that, at any rate in certain cases, they can be perceived; but speaking for myself I feel confident that this is not true. I do not perceive them... moreover I shrewdly suspect that others do not perceive them either, because they are able to come to so very little agreement as to which of them relates any two given propositions. (p. 161)

For example, for even very simple pairs of propositions (such as 'This is red' and 'That is blue'), where one might have expected objective relations to be more readily accessible, there is very little agreement as to what probability relation might connect them. Ramsey also notes that, while most people will agree that the probability of a fair coin landing heads is $\frac{1}{2}$, 'we can none of us say exactly what is the evidence which forms the other term for the probability relation about which we are then judging' (p. 162).

Having dismissed Keynes' theory, Ramsey moves on to his alternative, subjectivist view. The guiding idea throughout is that:

> [T]he degree of a belief is a causal property of it, which we can express vaguely as the extent to which we are prepared to act on it. (p. 169)

That is, partial beliefs are connected to choice and action, in that the more confidence one has in a proposition p, the more willing one will be to choose those options that lead to favourable outcomes under the assumption that p is true. Although he does not say much about the nature of this connection in 'Truth and Probability,' in 'Facts and Propositions' (1931b) Ramsey expresses sympathy for the pragmatist view according to which a full belief that p just *is* a set of actions that tends to lead to favourable outcomes at worlds where p is true:

> It is, for instance, possible to say that a chicken believes a certain sort of caterpillar to be poisonous, and mean by that merely that it abstains from eating such caterpillars on account of unpleasant experiences connected with them. The mental factors in such a belief would be parts of the chicken's behaviour... Thus any set of actions for whose utility p is a necessary and sufficient condition might be called a belief that p... (p. 144)

It is plausible that Ramsey intended a similar account for partial beliefs—that partial beliefs either *are* patterns of behaviour, or perhaps behavioural dispositions, or that they're otherwise definable primarily in terms of such things. In any case, Ramsey recognised that, if this vague connection between belief and choice could be suitably precisified, then it could be used to build a definition of degrees of belief in terms of choices.

Like other quantities throughout the sciences, Ramsey argues, 'the degree of a belief... has no precise meaning unless we specify more exactly how it is to be measured' (p. 167). To precisify his account, therefore, Ramsey sketches a procedure for the measurement of partial belief, which includes as a part also the measurement of utilities. The procedure takes as input the subject's preferences (as revealed by her choices) over

(i) propositions that are maximally specific with respect to matters the subject cares about, which we will refer to henceforth as *worlds*,

(ii) *binary gambles* of the form 'world ω_1 if p, and world ω_2 otherwise', and

(iii) *ternary gambles* of the form 'ω_1 if p & q, ω_2 if p & $\neg q$, and ω_3 otherwise',

and it outputs precise numerical representations of her partial beliefs and utilities. It can be roughly summarised as follows:

Elicit Preferences: Elicit the subject's preferences by offering her a sequence of choices between pairs of worlds and/or gambles.

Determine Utilities: Assuming the subject is an expected utility maximiser, use her preferences to determine her numerical utilities.

Define Beliefs: Define (a) the subject's degree of belief towards p in terms of her utilities for binary gambles involving p, and (b) her degree of belief towards p given q using her utilities for binary and ternary gambles involving p and q.

The following paragraphs will briefly discuss these three steps. More thorough treatments of Ramsey's procedure can be found in (Sahlin 1990), (Bradley 2001), and (Elliott 2017).

Regarding the first step, Ramsey says:

> [Suppose] that our subject has certain beliefs about everything; then he will act so that what he believes to be the total consequences of his action will be the best possible. If then we had the power of the Almighty, and could persuade our subject of our power, we could, by offering him options, discover how he placed in order of merit all possible courses of the world... Suppose next that the subject is capable of doubt; then we could test his degree of belief in different propositions by making him offers of the following kind. Would you rather have world [ω_1] in any event; or world [ω_2] if p is true, and world [ω_3] if p is false? (p. 177)

A common objection to this early stage of the procedure is that if the experimenter were to convince their subject that they 'had the power of the Almighty', then this would radically alter the beliefs supposedly being measured (e.g., Jeffrey 1983, pp. 158–60; cf. Sobel 1998, pp. 255–6; Bradley 2001, §3.2; Eriksson & Rabinowicz 2013).

More prosaically, a measurement procedure can be considered accurate only if it doesn't itself signficantly alter the measurand. Measurement procedures usually involve *some* unavoidable change to the quantity being measured—placing a cold thermometer into a hot liquid will slightly cool the liquid, for example, but for most purposes this effect is negligible. The objection at hand is therefore that merely having the subject *come to believe* that they have some choice over these worlds and gambles will generally involve a *substantial* change to their beliefs—enough to undermine the accuracy of Ramsey's procedure. For example, the process might alter the subject's beliefs regarding the claim 'Experimenters have the capacity to make it the case that ω_1 obtains if p is true, and ω_2 obtains otherwise', with potentially many ripple on effects through to her other beliefs. Interestingly, Ramsey makes note of a similar problem for an alternative gambling-based measurement procedure (p. 170), but for unknown reasons does not discuss the worry as it arises for his own proposal.

The second step of Ramsey's procedure is the most complicated, and it requires a suite of background assumptions about the partial beliefs, utilities, and preferences of the subject. In his words,

> I propose to take as a basis a general psychological theory, which is now universally discarded, but nevertheless comes, I think, fairly close to the truth in the sort of cases with which we are most concerned. I mean the theory that we act in the way we think most likely to realize the objects of our desires, so that a person's actions are completely determined by his desires and opinions. (p. 174)

Ramsey's point here is that, given some general assumptions, it will be possible—at least sometimes—to specify *exactly* what a subject's degrees of belief and utilities are once we have enough information about her preferences. However, Ramsey's statement of the 'general psychological theory' in the quoted passage underspecifies the assumptions he implicitly relies on to justify his procedure. We can precisify matters on his behalf, and assume that for the subject in question,

A1. There exists a real-valued function $\mathcal{U}$ that represents the subject's utilities on an interval scale.[1] Furthermore, if p and q are logically equivalent, then $\mathcal{U}(p) = \mathcal{U}(q)$.

A2. There exists a real-valued function $\mathcal{P}$ that represents the subject's partial beliefs, and satisfies the laws of probability.[2]

A3. The utility of a gamble 'ω_1 if p, ω_2 if $\neg p$' is equal to its expected utility,

$$\mathcal{U}(\omega_1 \ \& \ p)\cdot\mathcal{P}(p) + \mathcal{U}(\omega_2 \ \& \ \neg p)\cdot\mathcal{P}(\neg p),$$

and likewise for ternary gambles, *mutatis mutandis.* That is, a gamble's utility is a weighted average of the utilities of its outcomes, the weights provided by their probabilities.

While these assumptions clearly involve some degree of idealisation, Ramsey justifies them by noting that the development of any measurement process 'cannot be accomplished without introducing a certain amount of hypothesis or fiction' (p. 168), and that the assumptions come close enough to the truth to render them still useful (p. 173).

The reasoning by which we go from preferences to a numerical representation of utilities is complicated, and in Ramsey's paper mostly left unstated. It begins with the notion of an *ethically neutral proposition*, which can here be defined as:

Ethical Neutrality. p is *ethically neutral* for a subject iff, for all worlds ω consistent with p and $\neg p$, the subject is indifferent between ω, $\omega \& p$, and $\omega \ \& \ \neg p$.

That is, a proposition p is ethically neutral if the truth or falsity of p is a matter of indifference to the subject, regardless of the wider context in which its truth or falsity obtains. Having characterised ethical neutrality in terms of preferences, Ramsey is ultimately able to construct a definition (in terms of preferences over gambles) for when the *difference in utility* between two worlds ω_1 and ω_2 *is equal to* that between ω_3 and ω_4.[3]

This provides Ramsey with the resources needed to sketch a *representation theorem*, which forms the centrepiece of his paper. According to this theorem, the subject's preferences will satisfy eight relatively simple conditions only if there exists at least one function, $\mathcal{U}$, such that the difference in utility between ω_1 and ω_2 is at least as great as that between ω_3 and ω_4 if and only if

$$\mathcal{U}(\omega_1) - \mathcal{U}(\omega_2) \ \geq \ \mathcal{U}(\omega_3) - \mathcal{U}(\omega_4).$$

Moreover, this function $\mathcal{U}$ is unique up to positive linear transformation—that is, for any other function $\mathcal{U}^*$ with the same property,

$$\mathcal{U}^*(x) = \mathcal{U}(x)\cdot r + c,$$

for some positive real number r and constant c. Given Ramsey's background assumptions A1–A3, these two points together entail that the subject's preferences satisfy Ramsey's conditions on preferences only if $\mathcal{U}$ is the function that represents her utilities on an interval scale. Or, to put the point more simply: *if* the subject's preferences satisfy Ramsey's conditions, *then* they contain enough information to determine her utilities exactly.

Included amongst Ramsey's axioms are some obvious coherence conditions, such as that preferences ought to be transitive; as well as less obvious conditions, e.g., that for every pair of worlds ω_1, ω_2, there's a ω_3 whose utility is exactly halfway between that of ω_1 and ω_2. There has been very little empirical investigation into Ramsey's axioms, primarily due to the widespread opinion that his representation result has been superseded by the results of Savage (1954) and later decision theorists (see, e.g., Fishburn 1981). However, Ramsey's assumption that there exists an ethically neutral proposition (needed for the definition of equal differences in utility to make sense) has attracted substantial critical discussion. Ramsey provides his readers with no reasons to believe that even one ethically neutral proposition exists, still less that their existence is a precondition for the *consistency* of partial beliefs. For discussion, see (Sobel 1998), (Bradley 2001), (Eriksson & Hájek 2007), and (Elliott 2017).

The third and final stage of Ramsey's measurement procedure takes us from the subject's utilities to a definition of her degrees of belief:

> Having thus defined a way of measuring value we can now derive a way of measuring belief in general. If the option of [ω_2] for certain is indifferent with that of ['ω_1 if p, ω_3 otherwise'], we can define the subject's degree of belief in p as the ratio of the difference between [ω_2] and [ω_3] to that between [ω_1] and [ω_3]. (p. 179)

In a footnote, Ramsey adds that ω_1 must imply p, and ω_3 must imply $\neg p$. The definition makes sense in light of the background assumptions A1–A3 above: the world ω_2 has the same utility as the gamble 'ω_1 if p, ω_3 otherwise' just in case

$$\mathcal{U}(\omega_2) \;=\; \mathcal{U}(\omega_1)\cdot\mathcal{P}(p) \,+\, \mathcal{U}(\omega_3)\cdot(1 \,-\, \mathcal{P}(p))$$

Where $\mathcal{U}(\omega_1) \neq \mathcal{U}(\omega_3)$, this equality can be rearranged to give a definition of $\mathcal{P}(p)$:

$$\mathcal{P}(p) \;=\; \frac{\mathcal{U}(\omega_2) - \mathcal{U}(\omega_3)}{\mathcal{U}(\omega_1) - \mathcal{U}(\omega_3)}$$

As Ramsey notes,

> This amounts roughly to defining the degree of belief in p by the odds at which the subject would bet on p, the bet being conducted in terms of differences of value as defined. (pp. 179–80)

Ramsey's definition of conditional probability (p. 180) follows an essentially similar strategy.

Having thus outlined his strategy for defining degrees of belief, Ramsey proves that from his preference conditions and subsequent definitions, the following 'laws of probability' follow:

1. $\mathcal{P}(p) + \mathcal{P}(\neg p) = 1$
2. $\mathcal{P}(p \text{ given } q) + \mathcal{P}(\neg p \text{ given } q) = 1$
3. $\mathcal{P}(p \mathbin{\&} q) = \mathcal{P}(p) \cdot \mathcal{P}(q \text{ given } p)$
4. $\mathcal{P}(p \mathbin{\&} q) + \mathcal{P}(p \mathbin{\&} \neg q) = \mathcal{P}(p)$

These four conditions then imply that $\mathcal{P}$ satisfies *finite additivity*:

5. Where $p \mathbin{\&} q$ is impossible, $\mathcal{P}(p \text{ or } q) = \mathcal{P}(p) + \mathcal{P}(q)$

Ramsey goes on to say:

> These are the laws of probability, which we have proved to be necessarily true of any consistent set of degrees of belief. Any definite set of degrees of belief which broke them would be inconsistent in the sense that it violated the laws of preference between options... (p. 182)

In this passage we find an early version of a representation theorem argument for probabilistic norms on partial belief and for expected utility theory, of the kind later made popular by Savage. (See also Skyrms 1987, and the entry on Normative Theories of Rational Choice: Expected Utility, §2.2). It is dubious, however, that Ramsey really proved that the stated laws must be true of any consistent set of partial beliefs. Amongst the axioms of his representation theorem for utilities are several non-necessary conditions, as well as conditions (like those regarding the ethically neutral proposition) which are not plausible as conditions of *consistency* on preferences. More importantly, Ramsey's 'proof' is grounded in strong theoretical assumptions about the connection between degrees of belief and preferences, which Ramsey admits are idealizations.

In the same paragraph, Ramsey states that:

> If anyone's mental condition violated these laws, his choice would depend on the precise form in which the options were offered him, which would be absurd. He could have a book made against him by a cunning better and would then stand to lose in any event. (p. 182)

The reasoning behind the last claim is never made explicit, though it is evident that Ramsey was putting forward what has come to be known as a Dutch Book Argument, later made more precise by de Finetti (1937; see entry the on Dutch Book Arguments).

Having established what he takes to be the conditions of *consistency* for partial beliefs, Ramsey concludes his paper with a lengthy discussion on what (in addition) might make a set of partial beliefs *reasonable*. He proposes a condition of *calibration*, or fit with known frequencies:

> Let us take a habit of forming opinion in a certain way; e.g. the habit of proceeding from the opinion that a toadstool is yellow to the opinion that it is unwholesome. Then we can accept the fact that the person has a habit of this sort, and ask merely what degree of opinion that the toadstool is unwholesome it would be best for him to entertain when he sees it... And the answer is that it will in general be best for his degree of belief that a yellow toadstool is unwholesome to be equal to the proportion of yellow toadstools which are in fact unwholesome. (This follows from the meaning of degree of belief.) (p. 195)

So, for example, if 1 in 100 yellow toadstools is unwholesome, then *ceteris paribus* one should believe that *this* toadstool is unwholesome, given that it is yellow, to degree 0.01. Something like this condition has reappeared in a number of later works (e.g., Shimony 1988; Lewis 1980); see the entry on Interpretations of Probability, §3.3.4, for more discussion.

It would be hard to understate the importance of the above ideas to the subjectivist tradition. It is a major testament to the originality of Ramsey's essay that it contains not only the first appearances of two of the main contemporary arguments for probabilistic norms on partial belief, but also an influential case for a normative link between partial beliefs and known frequencies. Furthermore, most attempts to characterise degrees of belief in the subjectivist tradition have made central appeal to their connection with preferences. (See de Finetti 1937, Savage 1954, Anscombe & Aumann 1963, Maher 1993; for criticisms of the approach, see Joyce 1999, §1.3; Eriksson & Hájek 2007.) Indeed, in much of philosophy, economics and psychology today, the default or orthodox way to operationalise degrees of belief is in terms of choices, along essentially the same lines that Ramsey put forward.

Notes

[1] An *interval scale* is a numerical scale in which intervals of differences are meaningful. For instance, if $\mathcal{U}(p) - \mathcal{U}(q) = \mathcal{U}(r) - \mathcal{U}(s)$, then the difference in utility between p and q is equal to the difference between r and s. It doesn't follow, on the other hand, that if $\mathcal{U}(p) = 2{\cdot}\mathcal{U}(q)$, then p has *twice* as much utility as q; this is because the 'zero' point in an interval scale is arbitrary. Examples of interval scales include temperatures as measured in degrees Celsius or Fahrenheit, and yearly dates as measured in the A.D. and Bhuddist or Hindu systems.

[2] Specifically, Ramsey's definition of unconditional probabilities presupposes that $\mathcal{P}$ satisfies $\mathcal{P}(p) = 1 - \mathcal{P}(\neg p)$. (See note 3 for more details.) Furthermore, his definition of conditional probabilities presupposes that $\mathcal{P}(p$ given $q) = \mathcal{P}(p \,\&\, q)/\mathcal{P}(q)$ whenever $\mathcal{P}(p) > 0$, and $\mathcal{P}(p$ given $q) = 1 - \mathcal{P}(\neg p$ given $q)$.

[3] The derivation proceeds as follows. Where p is ethically neutral and the subject not indifferent between ω_1 and ω_2, $\mathcal{P}(p) = 0.5$ whenever the subject is indifferent between 'ω_1 if p, ω_2 otherwise' and 'ω_2 if p, ω_1 otherwise'. To see this, note that under A1–A3, the indifference holds iff

$$\mathcal{U}(\omega_1 \,\&\, p)\cdot\mathcal{P}(p) + \mathcal{U}(\omega_2 \,\&\, \neg p)\cdot\mathcal{P}(\neg p) = \\ \mathcal{U}(\omega_2 \,\&\, p)\cdot\mathcal{P}(p) + \mathcal{U}(\omega_1 \,\&\, \neg p)\cdot\mathcal{P}(\neg p).$$

Assuming that p is ethically neutral, this can be re-written:

$$\mathcal{U}(\omega_1)\cdot\mathcal{P}(p) + \mathcal{U}(\omega_2)\cdot\mathcal{P}(\neg p) = \\ \mathcal{U}(\omega_2)\cdot\mathcal{P}(p) + \mathcal{U}(\omega_1)\cdot\mathcal{P}(\neg p),$$

where $\mathcal{U}(\omega_1) \neq \mathcal{U}(\omega_2)$. Given A2, we can then further reduce the equality:

$$\mathcal{U}(\omega_1)\cdot\mathcal{P}(p) + \mathcal{U}(\omega_2)\cdot(1 - \mathcal{P}(p)) = \\ \mathcal{U}(\omega_2)\cdot\mathcal{P}(p) + \mathcal{U}(\omega_1)\cdot(1 - \mathcal{P}(p)).$$

From there we can derive by that $\mathcal{P}(p) = 0.5$.

Now suppose p is an ethically neutral proposition of probability 0.5, and that the subject is indifferent between 'ω_1 if p, ω_4 if $\neg p$' and 'ω_2 if p, ω_3 if $\neg p$'. This holds iff

$$\mathcal{U}(\omega_1 \,\&\, p)\cdot\mathcal{P}(p) + \mathcal{U}(\omega_4 \,\&\, \neg p)\cdot\mathcal{P}(\neg p) = \\ \mathcal{U}(\omega_2 \,\&\, p)\cdot\mathcal{P}(p) + \mathcal{U}(\omega_3 \,\&\, \neg p)\cdot\mathcal{P}(\neg p),$$

which we can now quickly reduce to

$$\mathcal{U}(\omega_1)\cdot\mathcal{P}(p) + \mathcal{U}(\omega_4)\cdot(1 - \mathcal{P}(p)) = \\ \mathcal{U}(\omega_2)\cdot\mathcal{P}(p) + \mathcal{U}(\omega_3)\cdot(1 - \mathcal{P}(p)).$$

Since $\mathcal{P}(p) = 1 - \mathcal{P}(p) = 0.5$, we can drop out the constant factor, leaving us with

$$\mathcal{U}(\omega_1) + \mathcal{U}(\omega_4) = \mathcal{U}(\omega_2) + \mathcal{U}(\omega_3),$$

which holds just in case

$$\mathcal{U}(\omega_1) - \mathcal{U}(\omega_2) = \mathcal{U}(\omega_3) - \mathcal{U}(\omega_4).$$

Since we've assumed that $\mathcal{U}$ represents the subject's utilities on an interval scale, it follows that the difference in utility between ω_1 and ω_2 is equal to that between ω_3 and ω_4 whenever (i) assumptions A1–A3 hold, and (ii) the subject is indifferent between the gambles 'ω_1 if p, ω_4 otherwise' and 'ω_2 if p, ω_3 otherwise', for some ethically neutral proposition p of probability 0.5.

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